A Note on Heat Transfer to Turbulent Liquid Falling Films at High Prandtl Number

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INTRODUCTION

Virtually all previous analyses of turbulent heat transfer in falling liquid films have used approximate eddy diffusivity or mixing length models to predict the resulting transport. Analyses of this type have been developed for film evaporation and condensation (Seban, 1954; Dukler, 1960; Lee, 1964; Kunz and Yerazunis, 1969; Mills and Chang, 1973; Seban and Faghri, 1976; Mostofizadeh and Stephan, 1981; Kutaleladze, 1982) as well as for simple heating and cooling of the liquid film (Seban and Faghri, 1976; Limberg, 1973).

While analysis of this type of transport has been the object of considerable effort, the heat transfer in the limit of high Prandtl number, Pr, has received much less attention. This is partly due to the fact that evaporation and condensation processes rarely involve liquid films with Prandtl numbers greater than 10. However, heating and cooling of liquid films in chemical or food processing may often involve organic liquids with high Prandtl number.

In a recent study of heat transfer across turbulent falling liquid films, Sandall et al. (1984) analytically determined an expression for the heat transfer coefficient using the assumption of high Prandtl number. Their heat transfer correlation was found to agree well with the evaporation data of Chun and Seban (1971). Heating or cooling of the film itself was not specifically considered.

For heating of turbulent falling films at large Pr, the resistance to heat transfer exists almost entirely in the viscous sublayer near the wall, with the outer fully turbulent region being virtually isothermal. In this note, an approximate relation for the heat transfer coefficient is derived for heating or cooling of a fully turbulent falling film at high Prandtl number. The analysis used to determine the heat transfer correlation incorporates the high Prandtl number assumption in a manner similar to that of Sandall et al. (1984). However, the present analysis differs in that it accounts for the known variation of turbulent Prandtl number with distance from the wall. Sandall et al. (1984) assumed that the turbulent Prandtl number is constant across the film. The heat transfer coefficients predicted by the correlation obtained here are found to agree well with experimental data for heating of liquid films at moderate and high values of Prandtl number.

ANALYSIS AND DISCUSSION

For turbulent flow at high Prandtl number, a steep temperature gradient is expected to exist in the viscous sublayer, with the outer region of the film being nearly isothermal. In the present analysis, it is therefore assumed that the temperature varies only in the viscous sublayer, with the outer region at a uniform temperature, T_i. Heat transfer from the liquid film to the surrounding gas is assumed to be negligible. In the thin viscous sublayer, convection of thermal energy is assumed to be negligible compared with diffusion, so the time-averaged boundary layer form of the energy equation in this region becomes

$$\frac{\partial}{\partial y}\left(\alpha \frac{\partial T}{\partial y} - \overline{v'T'}\right) = 0 \qquad (1)$$

Defining the eddy diffusivity of thermal energy in the usual manner and substituting and integrating Eq. 1 across the viscous sublayer yields

$$\frac{(T_w - T_i)k\sqrt{\tau_o/\rho_f}}{q''\nu} = \int_0^{\delta_v^+} \frac{dy^+}{1 + \dfrac{Pr\,\epsilon_M(y+)}{Pr_t\,\nu}} \qquad (2)$$

where T_w and T_i are the wall and interface temperatures. Note that for heating and cooling of the film, T_i is equal to the mean temperature of the film, T_m, since the temperature in the outer region is virtually uniform at a given downstream location. A negligible amount of mass is in the sublayer, where the temperature does vary, so the mean temperature is essentially equal to T_i.

If Pr is large, the integrand in Eq. 2 is small, except where ϵ_M is sufficiently small that $Pr\epsilon_M/\nu = 0(1)$. Thus, to evaluate the integral in Eq. 2, an accurate relation is needed for ϵ_M only in the near wall region. Studies by Tien (1964) and Simonek (1983) indicate that ϵ_M/ν is proportional to $(y+)^3$ at small y^+. It was therefore assumed that

$$\epsilon_M/\nu = \gamma(y+)^3 \qquad (3)$$

To evaluate Pr_t, the relation proposed by Kays and Crawford (1980) was used here:

$$Pr_t(\eta) = \left[\frac{1}{2\,Pr_{t\infty}} + \frac{C\eta}{\sqrt{Pr_{t\infty}}} - C^2\eta^2 \times \left[1 - \exp\left(\frac{-1}{C\eta\sqrt{Pr_{t\infty}}}\right)\right]\right]^{-1} \qquad (4)$$

where

$$C = 0.2, \quad Pr_{t\infty} = 0.86, \quad \eta = \frac{\epsilon_M}{\nu} Pr \qquad (5)$$

With Eq. 3, Eq. 2 can be written as

$$\frac{h}{k}\left(\frac{\nu}{\sqrt{\tau_o/\rho_f}}\right) = \frac{(\gamma Pr)^{1/3}}{I} \qquad (6)$$

where

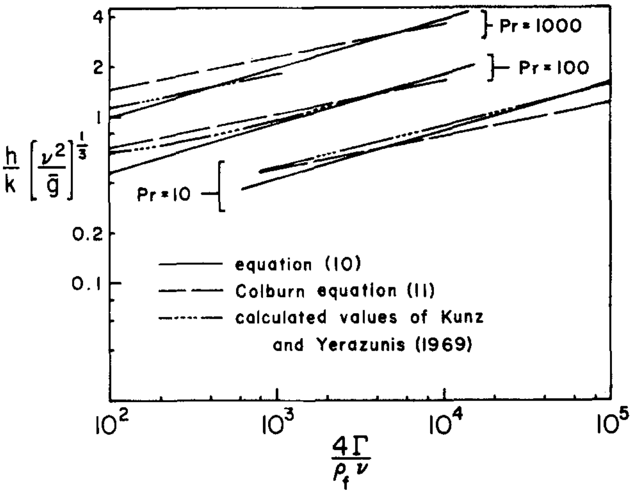

Figure 1. Comparison of the local Nusselt number predicted by the present analysis with those predicted by the analysis of Kunz and Yerazunis (1969) and Colburn Eq. 11.

$$I = \frac{1}{3}\int_0^{\gamma(\delta_0^+)^3 Pr} \frac{\eta^{-2/3}}{1 + \eta/Pr_t(\eta)} d\eta \tag{7}$$

In the limit of large Pr, the upper limit of the integral (Eq. 7) becomes large. Since the integrand rapidly becomes small at large η anyway, here the limit was taken to be $\eta = \infty$. Numerical integration of Eq. 7 using Eqs. 4 and 5 then yielded $I = 1.253$.

The integral I given by Eq. 7 can be evaluated by using Eq. 4 for Pr_t or with a constant value of Pr_t. Kays and Crawford (1980) indicate that Pr_t varies from about 0.86 away from the wall to 1.8 very near it. Since the region very near the wall is most important here, a choice of $Pr_t = 1.8$ might be appropriate. On the other hand, a constant value of $Pr_t = 0.9$ has been most often used in other studies of falling film heat transfer at moderate Prandtl numbers. At large Prandtl number, the proper choice of a constant Pr_t value is not clear a priori. Use of Eq. 4 circumvents this difficulty and should provide a physically realistic prediction of the heat transfer for these conditions.

For choices of constant Pr_t between 0.86 and 1.8, the value of I will vary by about 30%. Since the heat transfer coefficient is inversely proportional to I, it is in fact sensitive to the manner in which Pr_t is evaluated.

Convection of momentum in the film is neglected here, which is consistent with the assumption of a thin liquid film. Integration of the time-averaged u momentum equation across the film then yields a simple balance between the wall shear stress and the gravity force on the liquid

$$\frac{\tau_o}{\rho_f} = g\frac{(\rho_f - \rho_g)}{\rho_f}\,\delta \sin\theta \tag{8}$$

where θ is the angle between the surface and the horizontal.

For simplicity, the velocity profile in the film was taken to be the power law formula for turbulent flow near a wall, as suggested by Kays and Crawford (1980).

$$u^+ = 8.75\,(y^+)^{1/7} \tag{9}$$

In the present analysis, γ was taken to be 6.8×10^{-4}, the value indicated by Kato et al. (1968). This value of γ is also consistent with the range of values indicated by the results of Simonek (1983) for pipe flows. With this value of γ, Eqs. 7, 9, and 10 are easily combined to yield the following relation for the local heat transfer coefficient:

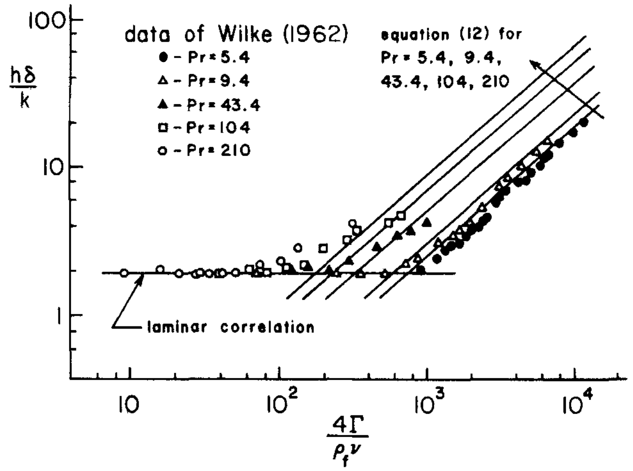

Figure 2. Comparison of the Nusselt number variation predicted by the present analysis with the data of Wilke (1962).

$$\frac{h}{k}\left(\frac{\nu^2}{g}\right)^{1/3} = 0.0259 Pr^{1/3} Re^{7/24} \tag{10}$$

Equation 10 is compared with the numerically calculated results of Kunz and Yerazunis (1969) for falling films at high Prandtl number in Figure 1. Also shown are curves predicted by the often-quoted Colburn (1933) equation:

$$\frac{h}{k}\left(\frac{\nu^2}{g}\right)^{1/3} = 0.056 Pr^{1/3} Re^{0.2} \tag{11}$$

As seen in Figure 1, the results of Eq. 10 compare favorably with the results predicted by the full numerical analysis and the Colburn relation (Eq. 11), even at $Pr = 10$.

The correlation obtained by Sandall et al. (1984) for heat transfer across a turbulent film at high Prandtl number may be modified to apply to heating of the film by setting some of the coefficients in their equation to zero. The correlation thus obtained predicts heat transfer coefficients significantly different from those predicted by Eq. 10. For example, at $Pr = 100$ and Re between 300 and 10^4, their equation predicts coefficients as much as 27% higher than Eq. 10. The effect of Pr_t on I noted above suggests that the larger h values are at least partly due to the assumed constant value of $Pr_t = 0.9$ in their analysis.

Wilke (1962) experimentally measured the local heat transfer coefficients for heating of water-ethylene glycol films falling on the outside of a vertical tube. The results were presented in terms of a Nusselt number based on the local film thickness, Nu_δ. Using the relations given above, one can rearrange Eq. 10 to obtain

$$Nu_\delta = \frac{h\delta}{k} = 0.00352 Pr^{1/3} Re^{7/8} \tag{12}$$

The predicted results of Eq. 12 are compared with the data of Wilke (1962) in Figure 2. Note that Nu_δ is approximately 2 for the fully laminar condition, because then $T_w - T_m \simeq (T_w - T_i)/2$. Although only a few of the data at high Pr values appear to be in the fully turbulent range, the trend in the data is consistent with a gradual transition from the laminar flow behavior to the fully turbulent values predicted by Eq. 12. For Pr values of 9.4 and 5.4, at high Reynolds numbers, agreement is quite good.

In summary, the heat transfer correlations obtained with the approximate analysis described above are in good agreement with the full analysis of Kunz and Yerazunis (1969) and the experimental data of Wilke (1962). These correlations appear to provide a simple, yet accurate means of predicting heat transfer for these conditions. In addition, the success of the approximate analysis presented here seems to confirm the assumptions made in the analysis regarding the nature of the transport in falling turbulent films at high Prandtl number.

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NOTATION

g = gravitational acceleration
$\bar{g}$ = $g \sin\theta(\rho_f - \rho_g)/\rho_f$
h = local heat transfer coefficient
I = integral defined by Eq. 8
k = thermal conductivity
Nu_δ = $h\delta/k$
Pr = Prandtl number
Pr_t = turbulent Prandtl number, ϵ_M/ϵ_H
q'' = surface heat flux
Re = film Reynolds number, $4\Gamma/\rho_f \nu_f$
T = mean local temperature
T' = temperature fluctuation
u = mean local velocity in the film parallel to the surface
u^+ = $u/\sqrt{\tau_o/\rho_f}$
v' = fluctuation in velocity component in film normal to surface
y = distance normal to surface
y^+ = $y\sqrt{\tau_o/\rho_f}/\nu$

Greek Letters

α = thermal diffusivity
γ = constant in Eq. 4
Γ = mass flow in film per unit width of surface, $\rho_f \int_0^\delta u dy$
δ = film thickness
δ_v = thickness of viscous sublayer
δ^+ = $\delta\sqrt{\tau_o/\rho_f}/\nu$
ΔT = temperature difference between the heated wall and the liquid-gas interface, $T_w - T_i$
ϵ_H = eddy diffusivity of heat, $-\overline{v'T'}/(\partial T/\partial y)$
ϵ_M = eddy diffusivity of momentum
ν = kinematic viscosity of liquid
ρ_f = density of liquid
ρ_g = density of surrounding gas
τ_o = shear stress at surface

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